“Necking” is observed in most tensile tests on metals. It’s an instability that arises when the work (strain) hardening rate of the metal is no longer sufficient to counter the tendency towards localization of the strain.

A generalized treatment of such instabilities was formulated by Considère well over a century ago. The factors affecting its onset are therefore well established, but detailed interpretation of stress-strain curves in the post-necking regime is complex, and frequently misunderstood. However, FEM modeling does allow various insights into the behavior in this regime, with potential for revealing information (about the final fracture event, as well as post-necking plasticity) that is otherwise inaccessible.
1 NOMINAL AND TRUE STRESS-STRAIN PLOTS

An appreciation of the subtleties of necking involves the concept of nominal and true stresses and strains. The standard outcome of a tensile (or compression) test is a stress strain curve.

Such plots commonly extend up to relatively high (plastic) strains - at least a few % and commonly several tens of %. It is common practice to equate the stress to the force divided by the original sectional area and the strain to the change in length (along the loading direction) divided by the original length. These are "nominal" ("engineering") values. The true stress acting on the material at any stage is the force divided by the current sectional area. After a finite (plastic) strain, under tensile loading, this area is less than the original area, as a result of the lateral contraction needed to conserve volume, so that the true stress is greater than the nominal stress.

Consider a sample of initial length \( L_0 \), with an initial sectional area \( A_0 \). For an applied force \( F \) and a current sectional area \( A \), conserving volume, the true stress can be written as

\[
\sigma_T = \frac{F}{A} = \frac{F L}{A L_0} = \frac{F}{A_0} (1 + \epsilon_n) = \sigma_n (1 + \epsilon_n)
\]

where \( \sigma_n \), is the nominal stress and \( \epsilon_n \) is the nominal strain. If \( \epsilon_n \), is positive (tensile test), then \( \sigma_T \) is larger than \( \sigma_n \). Similarly, the true strain can be written as in Equation 2.

\[
\epsilon_T = \frac{dL}{L} = \ln \left( \frac{L}{L_0} \right) = \ln (1 + \epsilon_n)
\]

This formulation is such that, for positive \( \epsilon_n \) (tensile testing), the value of \( \epsilon_T \) is smaller than \( \epsilon_n \). If the strain exceeds a few %, then the differences between true and nominal values start to become significant. This is illustrated by Fig.1, which shows a true stress v. true (plastic) strain plot - in this case one exhibiting linear work-hardening - and the corresponding nominal stress v. nominal strain curve (obtained via Eqns. (1) and (2)). It is clear that, above a few %, the differences between the two are substantial.
The conversions are thus straightforward, although it is important to appreciate that they are only valid if the stress and strain fields within the sample (gauge length) are uniform (homogeneous) - which can only be true prior to the onset of necking. In practice, it is common to present only the nominal plot, and several procedures for extraction of key parameters are based only on inspection of such curves. However, if the objective is to obtain fundamental information about the plasticity (and failure) characteristics of the material, then it is a plot of true stress against true strain that provides this.

![Stress-strain plots, in true and nominal forms, for a metal with a yield stress of 300 MPa and a (linear) work hardening coefficient, K, of 1,000 MPa.](image)

Figure 1: Stress-strain plots, in true and nominal forms, for a metal with a yield stress of 300 MPa and a (linear) work hardening coefficient, K, of 1,000 MPa.
With a brittle material, tensile testing may give an approximately linear stress-strain plot, followed by fracture (at a stress that may be affected by the presence and size of flaws). However, most metals do not behave in this way and are likely to experience considerable plastic deformation before they fail. Initially, this is likely to be uniform throughout the gauge length. Eventually, of course, the sample will fail (fracture). However, in most cases, failure will be preceded by at least some necking. The formation of a neck is a type of instability, the formation of which is closely tied in with work hardening. It is clear that, once a neck starts to form, the (true) stress there will be higher than elsewhere, probably leading to more straining there, further reducing the local sectional area and accelerating the effect.

In the complete absence of work hardening, the sample will be very susceptible to this effect and will be prone to necking from an early stage. Work hardening, however, acts to suppress necking, since any local region experiencing higher strain will move up the stress-strain curve and require a higher local stress in order for straining to continue there. Generally, this is sufficient to ensure uniform straining and suppress early necking. However, since the work hardening rate often falls off with increasing strain, this balance is likely to shift and may eventually render the sample vulnerable to necking. Furthermore, some materials (with high yield stress and low work hardening rate) may indeed be susceptible to necking from the very start.
Instabilities of this general type are actually quite common: such a situation was originally analyzed by Armand Considère (1885) in the context of the stability of structures such as bridges. Instability (onset of necking) is expected to occur when an increase in the (local) strain produces no net increase in the load, $F$. This will happen when

$$\Delta F = 0$$  \hspace{1cm} (3)

this leads to

$$F = A\sigma, \quad \therefore dF = A d\sigma + \sigma dA = 0$$

$$\therefore \frac{d\sigma}{\sigma} = \frac{-dA}{A} = \frac{dL}{L} = d\varepsilon \quad \text{(4)}$$

$$\therefore \sigma = \frac{d\sigma}{d\varepsilon}$$

with the $T$ subscript being used to emphasize that these stresses and strains must be true values. Necking thus starts when the slope of the true stress / true strain curve falls to a value equal to the true stress at that point. Fig.2 shows the construction for a (true) stress-strain curve represented by a simple analytical expression (the Ludwik-Hollomon equation - see our article on constitutive laws for plasticity).

**Figure 2:** The Considère construction for prediction of the onset of necking, expressed in the form of Eqn.(4) and applied to a material exhibiting a Ludwik-Hollomon true stress – true strain curve with the parameter values shown.
Necking Phenomena during Tensile Testing

The condition can also be expressed in terms of the nominal strain:

\[
\frac{d\sigma_N}{d\varepsilon_N} = \frac{d\sigma_T}{d\varepsilon_T} \left( \frac{L}{L_o} \right) = \frac{d\sigma_T}{d\varepsilon_T} \left( 1 + \varepsilon_n \right)
\]

\[\therefore \text{ at the instability point, } \sigma_N = \frac{d\sigma_T}{d\varepsilon_T} (1 + \varepsilon_n)\]

It can therefore also be formulated in terms of a plot of true stress against nominal strain. On such a plot, necking will start where a line from the point \(\varepsilon_n = -1\) forms a tangent to the curve. This is shown in Fig.3, which was obtained using the same Ludwik-Hollomon representation of the true stress – true strain relationship as that of Fig.2.

Some materials with high yield stress and low work hardening rate may be susceptible to necking from the start.

\[\sigma_T = 460 \text{ MPa} \]
\[K = 1000 \text{ MPa} \]
\[n = 0.5\]

Figure 3: The Considère construction, expressed in the form of Eqn.(5) and applied to a material exhibiting a Ludwik-Hollomon true stress – true strain curve with the parameter values shown.

It is important to note that the condition also corresponds to a peak (plateau) in the nominal stress – nominal strain plot. This can be seen on obtaining the gradient of such a plot by differentiating \(\sigma_N\) with respect to \(\varepsilon_N\):

\[
\sigma_N = \frac{\sigma_T}{1 + \varepsilon_n}
\]

\[\therefore \frac{d\sigma_N}{d\varepsilon_N} = \frac{d\sigma_T}{d\varepsilon_T} \left( 1 + \varepsilon_n \right)^2 \left( 1 + \varepsilon_n \right)^{-2}
\]

Substituting for the true stress – nominal strain gradient (at the onset of necking) from Eqn.(5)

\[\frac{d\sigma_T}{d\varepsilon_T} = \frac{1}{(1 + \varepsilon_n)^2} \]
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The Considère construction has been successfully used for decades to explain the main features of necking (unstable for metals, but stable for some polymers). However, it is not a full description of what happens. For example, it takes no account of sample geometry: even for a cylindrical section, the precise way that the neck forms turns out to be dependent on the aspect ratio - i.e., the ratio of the uniform section length to its diameter. Such effects can, however, be simulated via FEM modeling.

It is sometimes stated that the initiation of necking arises from (small) variations in sectional area along the gauge length of the sample. However, in practice, for a particular metal, its onset does not depend on whether great care has been taken to avoid any such fluctuations. Furthermore, the introduction of such defects in an FEM model does not, in general, significantly affect the predicted onset. The (modeling) condition that does lead to necking is the assumption that, near the end of the gauge length, the sample is constrained from contracting laterally [1-3]. In practice, due to the increasing sectional area in that region, and because the material beyond the reduced section length will undergo little or no deformation, that condition is usually a fairly realistic one.

It may be noted at this point that it is common during tensile testing to extract the “strength”, in the form of an “Ultimate Tensile Stress” (UTS). This is usually taken to be the peak on the nominal stress v. nominal strain plot, which corresponds to the onset of necking, as outlined above. It should be understood that this value is not actually the true stress acting at failure. This is difficult to obtain in a simple way, since, once necking has started, the (changing) sectional area is unknown - although the behavior can often be captured quite accurately via FEM modeling – see below. Also, the “ductility” (or “failure strain”, or “elongation at failure”), often taken to be the nominal strain when fracture occurs, which is usually well beyond the strain at the onset of necking, does not correspond to the true strain in the neck when fracture occurs.

This point is illustrated by the plots [4] shown in Fig.4, which relate to a single material that was tensile tested with a range of values for the gauge length, which in this case was also the length of the reduced section part of the sample, and the diameter of the gauge section (which was circular). It can be seen that, while the behavior was similar for all samples up to the point of necking (peak in the plot), which was at about 6-8% strain for this material, the elongation to failure values cover a huge range, being larger for the samples with shorter gauge length and, with a given gauge length, for those with larger diameter (i.e., for those with a small aspect ratio).

The cause of this is simple. After the peak, with necking taking place, virtually all of the recorded elongation is due to straining in the neck. For shorter samples, this region constitutes a greater proportion of the gauge length, making the increase in (nominal) “strain” larger. Similarly, with a larger diameter, the contribution from necking is increased (for a given gauge length).

This effect can be clearly captured in an FEM model, as shown in Fig.4. Provided that the (true) stress-strain relationship is valid up to the high strains involved, and a suitable fracture criterion can be identified - a true (von Mises) plastic strain of 100% was used in the plots shown, then the complete (nominal) stress-strain curve, including the post-necking region, can be reliably predicted.

4 FEM MODELLING OF REAL NECKING BEHAVIOUR

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The constitutive law used here is that of Voce:

\[ \sigma_s = \sigma_s^0 - \left( \sigma_s^0 - \sigma_s^\infty \right) \exp \left( \frac{\varepsilon}{\varepsilon_0} \right) \]  

(8)

where \( \sigma_s^0 \) is a saturation stress, \( \sigma_s^\infty \) is the yield stress and \( \varepsilon_0 \) is a characteristic strain. In this way, for any true stress – true strain relationship, including an experimental one that cannot be expressed as an equation, FEM simulation can be used to predict the onset of necking. Further modeling outcomes are shown here for two metals, with the Voce law again used. The parameter values are typical of annealed and work-hardened copper [5]. Firstly, Fig.5 gives a pictorial indication of the state of these two samples soon after the onset of necking. This figure shows both photos of the samples and FEM-predicted sample shapes and fields of (axial) stress and strain within them. The AR-Cu exhibits little work hardening - it is initially in a work-hardened state - and necks at a nominal strain of about 15-20%. The stress and strain fields shown are starting to exhibit marked inhomogeneity, with levels of both rising in the neck region. The Ann-Cu exhibits more work hardening, resulting in a delay of necking up to about 30-35%.

A comparison is shown in Fig.6 between the measured and predicted (nominal) stress-strain curves for these two materials. A critical (von Mises) strain level was again used to determine the fracture point, with a value of 70% for the AR-Cu and 50% for the Ann-Cu. There is thus scope for inhomogeneity, with levels of both rising in the neck region. The Ann-Cu exhibits more work hardening, resulting in a delay of necking up to about 30-35%.

In summary, while the concept of the UTS is of at least some significance and value, particularly if considered in combination with the (true) stress-strain relationship during plastic deformation, the numbers obtained for the elongation at failure and the reduction in area are more or less meaningless. There is a strong argument for abandoning them entirely and concentrating on obtaining parameters that provide useful guidelines for assessment of the “strength” of a metal. On the other hand, provided the stress-strain relationship can be well-captured in a constitutive law (that holds up to relatively high strains), FEM simulation can be used to predict the complete (nominal) stress-strain curve. By comparing simulated and experimental curves in terms of the point at which fracture occurs, it may be possible to estimate the critical (von Mises) strain for fracture. This is a parameter that is widely used in FEM simulation of various practical situations, so being able to evaluate it for a particular material in this way is an attractive concept.
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REFERENCES


